

SCIENCE

NEW YORK, OCTOBER 2, 1891.

THE EVOLUTION OF ALGEBRA.¹

IN considering the possible subjects for an address on this occasion, it has seemed to me that a half-hour might be agreeably spent in a brief survey of the progress, or evolution, of algebra from its earliest known beginnings to the present time.

The realm of mathematics may be classified, in a general way, into (1) Arithmetic, or the theory of numbers, (2) Algebra, (3) Geometry, though sharp dividing lines cannot always be drawn between these departments: the last two, for instance, mutually interacting, geometry illustrating algebra, while algebra is the efficient servant of geometry, enabling it to conquer territory which it could scarcely have entered upon unaided.

The history of the development of these different branches of mathematics shows considerable diversities among them. Thus geometry reached in a short time, among the ancient Greeks, a high stage of advancement, and then became practically stationary until quite recent times, while the progress of algebra has been more in the nature of a gradual and continuous evolution. Nesselmann has recognized three stages in this development, which he designates as the rhetorical, the syncopated, and the symbolical, to which I may perhaps venture to add the "multiple," in which a plurality of fundamental units is recognized and treated. We may regard the first three as somewhat analogous to the stone, bronze, and iron ages in human history, overlapping each other, as do these, at different times and places; while the last may be compared to that age of aluminum which is perhaps dawning upon the world.

Rhetorical algebra was a process for determining the unknown quantity in an equation by a course of logical reasoning expressed entirely in words, without the use of any symbols whatever, similar to our present mental arithmetic. In course of time abbreviations of those words which constantly recurred were introduced, by the use of which the statement of the reasoning could be much shortened, it being even possible with the notation of Diophantos to approximate to the conciseness of the modern, or symbolic, method. This, however, was not done by Diophantos himself, who used his abbreviations strictly as such, and reasoned out his results in words combined with these. This method is what is designated by Nesselmann as the syncopated, and forms evidently a stepping-stone toward the symbolic, in which perfectly arbitrary symbols are employed to represent the various quantities dealt with, and no words are written out except a conjunction now and then.

The earliest traces of algebraic knowledge which have been discovered are found in Egypt, that wonderful land whose records carry us back to such a remote antiquity. Ahmes, in a papyrus manuscript, dating from about 1400 B.C., deals with certain geometric and algebraic problems, and seems to have had as good a conception of the symbolism of algebra as his successors of a much later period. Thus he had signs

for $+$, $-$, and $=$, and used the character representing a heap for the unknown quantity. He seems, therefore, to have long anticipated Diophantos in the use of syncopated notation. Our knowledge of Egyptian mathematics subsequent to this time is very slight, and is gleaned from the statements of various Greek and Latin authors.

We will pass, then, at once to the Greek contributions to the development of our subject. So far as can now be ascertained, probably but little strictly algebraic work was done before the third or fourth century of our era, though opinions differ on this point. The wonderful accomplishments of Archimedes were mainly geometrical and mechanical, though he makes one remark which is equivalent to a statement regarding the roots of an equation of the third degree, which is remarkable as being, with one exception, the only known case of any consideration of such an equation until after the lapse of more than a thousand years from his time.

Thymaridas in the second century of our era is the earliest mathematician known to have enunciated an algebraic theorem. This was, however, done entirely in words, no symbol for any quantity or operation being used.

Practically the foundation of algebra was laid by Diophantos of Alexandria. But little is known of this remarkable man. Though we have his writings in Greek, he was probably not himself a Greek. The period at which he lived is in dispute, though probabilities favor the fourth century of our era. Even the spelling of his name is uncertain, there being a question as to whether the last syllable should be *os* or *es*. But whatever may be known or unknown about the man himself, his writings show a very wonderful power of analytic reasoning, especially when we consider the awkwardness of the tools with which he was obliged to work.

What strikes us at once, from our present point of view, as most hampering is the fact that he had only one symbol for the unknown, so that, in dealing with a problem which would now be solved by the aid of several such symbols, as x , y , z , etc., he was obliged to adopt some expedient, such as to make mentally such combinations and arrangements as to get along with only one. It is easy to see how much ingenuity must often have been required to accomplish this. It is a curious and surprising fact that algebraic analysis was subjected to this same limitation down to a comparatively recent period. In place of the exponents at present used to indicate the powers to which quantities are raised, Diophantos designated the square and cube of the unknown by the initial letters of the corresponding words in Greek. Thus the unknown is represented by the character ς , standing for the word $\acute{\alpha}\rho\iota\theta\mu\acute{o}\varsigma$ (i.e., number), which is also frequently written out in full; the square of the same by δ° , a contraction for $\delta\acute{\upsilon}\nu\alpha\mu\iota\varsigma$ (power); and the cube by κ° , a contraction for $\kappa\acute{\upsilon}\beta\omicron\varsigma$ (cube). Higher powers up to the sixth were indicated by combination or repetition of these symbols. The origin of the character for *arithmos* is uncertain; it may be the final sigma of this word, or it may be a contraction of $\alpha\rho$, the first two letters of the same, or it may be derived from an old Egyptian symbol for the unknown. When oblique cases of these quantities are required, the words for square and cube are written out in full, while the practice varies with regard to *arithmos*, the word being sometimes

¹ Address before the Section of Mathematics and Astronomy of the American Association for the Advancement of Science, at Washington, D.C., Aug. 19-25, 1891, by E. W. Hyde, vice-president of the section.

written out, while at other times the case-termination is written above to the right, thus ζ^{ω} , the symbol being also generally doubled when the signification is plural.

Diophantos indicated addition merely by juxtaposition, having no sign for plus; for minus, however, he used the sign φ . As a consequence, in order to avoid confusion, he was obliged to do two things; first, to designate the absolute term as so many $\mu\omicron\nu\alpha'\delta\epsilon\varsigma$, or units, abbreviated into μ^{δ} , and second, to write all the negative terms together after the positive. Thus the quantity $x^3 - 5x^2 + 8x - 1$ would be written in Diophantos's notation.

$$\mu^{\delta} \tilde{\alpha} \zeta \zeta \alpha \tilde{\eta} \varphi \delta^{\delta} \tilde{\epsilon} \mu^{\delta} \tilde{\alpha}.$$

This may be rendered more expressive if we change it by substituting Arabic numerals, and putting U for units, N for number or unknown, S for square, and C for cube: thus it becomes $C1N8 - S5U1$.

It is to be noted that Diophantos and his successors up to comparatively recent times had no conception whatever of an intrinsically negative quantity as possible. Whatever sign may have been used for minus was considered as simply indicating that one number was to be subtracted from another, and if the subtrahend were larger than the minuend no meaning was attached to the expression.

It is possible that Diophantos might have been able to escape from the limitations of his system if the letters of the Greek alphabet had not been already appropriated for the representation of particular numbers, thus precluding their use as symbols of quantity in general.

It may be of interest to give at this point specimens of the purely rhetorical and of the syncopated methods of solution. They are given by Nesselmann, and are verbatim translations from the original tongues. The first is a solution of a quadratic equation by Mohammed ibn Musa, and the second the solution of a problem by Diophantos.

A square and ten of its roots are equal to nine-and-thirty units, that is, if you add ten roots to one square, the sum is equal to nine-and-thirty. The solution is as follows: halve the number of roots, that is, in this case, five; then multiply this by itself, and the result is five-and-twenty. Add this to the nine-and-thirty, which gives four-and-sixty; take the square root, or eight, and subtract from it half the number of roots, namely, five, and there remains three: this is the root of the square which was required and the square itself is nine.

(S = square, N = number, U = unit, as above.)

To divide the proposed square into two squares: Let it be proposed, then, to divide 16 into two squares; and let the first be supposed to be one square. Thus 16 minus one square must be equal to a square. I form the square from any number of N 's minus as many U 's as there are in the side of 16 U 's. Suppose this to be 2 N 's minus 4 U 's. Thus the square itself will be 4 squares 16 U 's minus 16 N 's. These are equal to 16 units minus 1 square. Add to each the negative term, and take equals from equals. Thus 5 squares are equal to 16 numbers. One (square) will be 25 twenty-fifths, and the other 144 twenty-fifths, and the sum of the two makes up 400 twenty-fifths, or 16 units, and each is a square.

Compare these long-drawn-out statements with their equivalents in modern notation:

First.	Second.
$x^2 + 10x = 39$	$16 - x^2 = \square = (2x - 4)^2$
$x^2 + 10x + 25 = 64$	$= 4x^2 + 16 - 16x$
$\therefore x + 5 = 8$	$\therefore 16x = 5x^2$
$\therefore x = 3$	$\therefore x = \frac{16}{5}$

The example from Diophantos evidently does not take full advantage of his notation, for the symbol for minus is not used, and in several cases the words are written out in full where abbreviations might have been employed. Further, no use is made of the symbol for equality, viz., $\dot{\iota}$, an abbreviation for $\dot{\iota}\sigma\omicron\iota$, which is elsewhere used by the author. If the fullest use of the syncopated notation had been made, the solution would have been somewhat comparable for conciseness and brevity with the modern method, only about twice as many characters and marks being required. Solutions in this abbreviated form appear on the margins of Diophantos's manuscripts, but they are believed to have been added by some one else, and not to be due to the author himself.

The works of Diophantos, called by him "Arithmetics," deal largely with indeterminate equations and the theory of numbers. Quadratic equations are constantly solved, but only real positive results are recognized or considered; and even when there are two positive roots, only one is taken account of. One very simple case of an equation of the third degree is found.

We will turn next to the consideration of the ancient algebra of India. There lived at Patna, in India, some time in the sixth century of our era, a mathematician named Arya-Bhatta, who wrote a work treating of arithmetic, algebra, geometry, trigonometry, and astronomy. It consists in the enunciation of rules and propositions in verse. The author gives, of course in a purely rhetorical manner, the sums of the first, second, and third powers of the first n natural numbers, the general solution of a quadratic equation, and the solution in integers of some indeterminate equations of the first degree.

The only other ancient Indian mathematician of prominence is Brahmagupta, who lived in the seventh century of our era. His work is also written in verse, and is called "Brahma-Sphuta-Siddhanta," or the "System of Brahma in Astronomy." Two chapters of this work deal with arithmetic, algebra, and geometry. The treatment of algebra is purely rhetorical, and includes a discussion of arithmetical progressions, quadratic equations (only the positive roots being considered), and indeterminate equations of the first degree, together with one of the second degree.

These Indian writings are of special interest as being the sources from which the Arabs derived their first knowledge of algebra. They obtained from the Greeks before A.D. 900, thorough translations of Euclid, Apollonius, Archimedes, and others, a knowledge of geometry, mechanics, and astronomy, but had no translation of Diophantos till a hundred and fifty years later, when they had themselves already made considerable progress in algebraic analysis. From the Arabians in turn western Europe obtained, not only the decimal notation of arithmetic, but also its first knowledge of other branches of mathematics.

The first great mathematician among the Arabs is generally known by the name of Alkarismi, though this is an incorrect transliteration of only one of his names. From the title of his work, "Al-gebr we'l Mukabala," we have the name of that branch of mathematics under consideration, al-gebr signifying that the same quantity may be added to or subtracted from both sides of an equation.

Alkarismi treats the quadratic, giving geometric proofs of rules for the solution of different cases, and recognizing the existence of two roots, though he only considers such as are real and positive. He treats only numerical equations, and no distinction is made between arithmetic and algebra. This

is true likewise of his Arabian successors, who, though they advanced so far as to obtain the general solution of a cubic equation, and to state such a proposition in integers of the equation $x^3 + y^3 = z^3$ is feasible, yet always adhered to the rhetorical method, and made scarcely any progress in general algebraic science. Indeed such progress was hardly possible until the introduction of symbolic methods.

The first decided steps in the direction of symbolism since the work of Diophantos were taken by a mathematician of India named Bhaskara in the twelfth century. He used abbreviations and initials to denote the unknown, a dot for minus, and juxtaposition to indicate addition. A product is denoted by the first syllable of the word for multiplication subjoined to the factors, division by the divisor being written beneath the dividend without a line between as our custom is now. The two sides of an equation are written one under the other, and explanatory records are introduced whenever it is necessary to prevent misunderstanding. Occasionally symbols are used for given as well as unknown quantities. Square, cube, and square root are denoted by the initial letters of the corresponding words. Using the Arabic, or decimal, notation, he has a character for zero, which enables him to write all his equations with all the powers of the unknown arranged in regular order on each side of the equation, certain of them being multiplied by the factor zero. This method of writing equations maintained itself till long afterwards. We have in this author a distinct advance over Diophantos and the Arabians in the introduction of various symbols for the unknown, so that several might be used in the same problem, as well as in the use of zero.

We have now to consider a new phase of algebraic progress arising from the introduction into western Europe of the works of the Arabian mathematicians. This took place through the Moors of Spain. The Greek and Arabic works were studied at the Moorish universities of Granada, Cordova, and Seville, but all knowledge of them was jealously kept from the outside world until the twelfth century, during which copies came into the possession of Christians. Up to this time Christian Europe had been almost a mathematical blank. The simple arithmetical operations they were able to perform were accomplished by the aid of the abacus, and they possessed some knowledge of astronomy and geometry, but made no progress until they were able to avail themselves of the previous labors of Greek, Hindu, and Arab, under the stimulus of which a career of advancement began which has continued to the present time. This career, however, did not begin immediately; it took several centuries to assimilate the material received from these sources, and thus to lay the foundations on which subsequent progress should rest.

During this period the rhetorical method was used in all algebraic processes, and it was not until the sixteenth century that syncopated methods were introduced, preparing the way for the symbolic methods that soon followed. Latin being the language in use, the word *res*, or *radix*, was employed for the unknown quantity, the square being called *census*, and the cube *cubus*. These words were at first written out in full and afterwards represented by *R* or *Rj*, *Z* or *C*, and *C* or *K* respectively.

The signs $+$ and $-$ are first found in a mercantile arithmetic by Johann Widmann, published in 1489, though they did not come into general use by mathematicians till a hundred years or more afterward. The most probable supposition as to their origin is that they were at first warehouse marks indicating an excess or deficiency in the contents of a

package which was supposed to contain a certain definite amount. Widmann uses them purely as abbreviations, not as symbols of operation.

The first mathematical work ever printed was by Pacioli, upon arithmetic, algebra, and geometry, and marks the beginning of the syncopated stage of development in western Europe. This book appeared in 1494, just before the beginning of the sixteenth century, during which this method was in vogue. Pacioli uses initials as abbreviations for the unknown, its square and cube, and for the words "plus" and "equal," also occasionally *de* for *demptus*, instead of minus.

The sign now used for equality was introduced by Recorde in an arithmetic published in 1540. He uses also the present signs $+$ and $-$. At about the same time our present symbol for square root was introduced by Stifel, and Nicholas Tartaglia discovered the solution of the cubic equation $x^3 + px = q$, which is generally attributed to Cardan, and goes by his name. Cardan obtained the solution from Tartaglia under promise of strict secrecy, and then published it in his work "Ars Magna." Considerable advance is made in this work over anything done by his predecessors. Negative and even imaginary roots of equations are discussed, and the latter are shown to always occur in pairs, though no interpretation of them is attempted. Cardan shows that when the roots of the cubic are all real, Tartaglia's solution appears in an imaginary form. This is the first notice we find of imaginaries, and, with the exception of a similar treatment by Bombelli a few years later, and a suggestion as to their interpretation by Wallis in 1685, they were discussed by no subsequent mathematician until Euler investigated them nearly two hundred years afterward. Cardan also discovered the relations between the roots and coefficients of an algebraic equation, and the underlying principle of Descartes' rule of signs. It is to be noted that his solutions both of quadratics and cubics are geometrical.

In 1572 Bombelli published an algebra in which the same subjects discussed by Cardan are treated in about the same way, but in which a marked advance is made in notation, viz., the employment for the unknown of the symbol 1 , while its powers are denoted by 2 , 3 , etc. Thus he would write $x^3 + 5x - 4$ as $1^2 p. 5 1 m. 4$, $p.$ and $m.$ standing for plus and minus. Other writers of the same period would have written the expression thus,

$$1Z p. 5R m. 4, \text{ or } 1Q + 5N - 4.$$

Up to this time in the development of algebraic notation, whatever may have been the forms or symbols used, they were regarded simply as abbreviations for the words necessary to express the idea to be conveyed. But now the conception of pure symbolism begins to appear. Vieta, who lived in the last half of the sixteenth century, denoted known quantities by consonants and unknown by vowels, while powers were indicated by initials or abbreviations of the words *quadratus* and *cubus*. He was thus enabled to deal with several unknowns in the same problem, together with their powers. The following is a specimen of his notation. The equation $3BA^2 - DA + A^3 = Z$ he writes as

$$B 3 \text{ in } A \text{ quad.} - D \text{ plano in } A + A \text{ cubo equatur} \\ Z \text{ solido.}$$

(It may be noted that he makes his equations homogeneous, and lays stress on the desirability of so doing.) This and the other examples that have been given above illustrate the great variety of notations in use during this period, no conventional system having yet been adopted to be adhered to in the main by all mathematical writers. This is, of course, an inevitable accompaniment of the formative stage of any

branch of science, when a few men are working here and there in comparative isolation. This variety continued to a considerable extent throughout the seventeenth century.

In this century we arrive at a new era in mathematical development. This was brought about by the application of algebra to geometry by Descartes in the early part, and the discovery of the differential calculus by Newton and Leibnitz independently in the latter part of the century. Algebra had been used in connection with geometry before Descartes, but to him was due the discovery of the fact, that, if the position of a point be given by co-ordinates, then any equation involving those co-ordinates will represent some locus all of whose properties are contained implicitly in the equation, and may be deduced therefrom by ordinary algebraic operations.

Descartes initiated the custom, which has become fixed, of using the first letters of the alphabet for known and the last for unknown quantities. He also appears to have been the first to perceive that one general proof is sufficient for any proposition algebraically treated, the different cases which might arise by different arrangements of the equations being covered by the possibility of any letter representing a negative as well as a positive quantity, i.e., he distinguished the intrinsic sign of a quantity or symbol. Hitherto it had been considered necessary to treat separately the forms of the quadratic $ax^2 + bx = c$, $ax^2 = bx + c$, etc., which was a natural result of the geometric method of arriving at the solution. Descartes also introduced our present notation for powers, taking his exponents, however, only as positive and integral.

Contemporaneously with Descartes, Cavalieri, in Italy, applied the so-called "method of indivisibles" to the computation of areas, volumes, etc., a process which gave way early in the eighteenth century to the integral calculus. At this time, also, the beginnings of the mathematical theory of probabilities were made by Pascal and Fermat in the solution of a certain problem which had been proposed.

A tremendous impulse was given to all branches of mathematics in the latter part of the seventeenth century by the genius of Newton. Besides his epoch-making discovery of the "theory of fluxions," or differential calculus, he contributed to algebraic science the idea of the general exponent or n th power (n being positive, negative, integral, or fractional), the binomial theorem, and a considerable part of the theory of equations.

To Leibnitz we owe the present notation of the differential calculus, the introduction of the terms "co-ordinates" and "axes of co-ordinates," and suggestions as to the use of indeterminate coefficients and determinants, which, though not developed by him, led, in the hands of others, to important results.

Jacob Bernoulli developed the fundamental principles of the calculus of probabilities, and made the first systematic attempts to construct an integral calculus. His brother John developed the exponential calculus, and treated trigonometry independently as a branch of analysis, it having been previously regarded as an adjunct of astronomy. The possibility of a calculus of operations was first recognized by Brook Taylor, after whom "Taylor's theorem" is named. De Moivre contributed to the discussion of imaginaries the important theorem which bears his name. In 1748 MacLaurin published an algebra which contained the results of some earlier papers published by him, among others one on the number of imaginary roots of an equation, and one on the determination of equal roots by means of the first derivative.

In the latter part of the eighteenth and beginning of the nineteenth centuries mathematical advancement was rapid under the powerful hands of Euler, Lagrange, Laplace, and Legendre. To these great men we owe the calculus of variations, the initial discussion of the calculus of imaginaries (which was afterwards systematized and developed by Gauss, Cauchy, and others), the treatment of determinants, contributions to the theory of equations, a large part of the integral calculus and differential equations, the development of the theory of probabilities, the treatment of elliptic functions, the method of least squares, and the specially algebraic treatment of the theory of numbers. In this list are included only those things which are of an algebraic nature.

We have now reached the beginning of our own century, in which the advance has been so rapid in all directions as to preclude more than a mere indication of some of the lines along which this has taken place, without any attempt at an enumeration of the illustrious names of those who have so magnificently carried forward the work.

The theory of equations has been perfected by the full use of the complex unit $a + bi$, forming thus, in the words of Cayley, a "universe complete in itself, such that, starting in it, we are never led out of it." We have, in fact, a double algebra as the instrument for the complete treatment of all higher analysis, except that in which one of higher multiplicity is used. The field of quantities has been brilliantly cultivated by Cayley, Sylvester, and others. The theory of matrices has been developed by Cayley, and it was shown by Professor J. Willard Gibbs, in his vice-presidential address before this section at the Buffalo meeting in 1886, that the simple and natural expression of this theory is in the language of multiple algebra. The φ of Hamilton is a matrix of the third order, and the Q of Grassmann a matrix of the n th order.

In the treatment of differential equations we have an algebra of operations, due primarily to George Boole, carried to a high degree of perfection, in which the symbol of differentiation is treated precisely as if it were a real quantity. In fact, we have come to regard scalar multiplication simply as a particular case in the calculus of operations which covers every possible case of the effect of one symbol upon another in producing some change in it. A further extension of this same idea we have in the algebra of logic, invented by the same author, and cultivated and extended by others since his time.

In conclusion, I propose to sketch briefly the development of the idea of a multiplicity of fundamental units, which is pervading more and more the mathematical thought of the day. This proceeded along two distinct lines, one arising from the interpretation of the imaginary, $\sqrt{-1}$, and the other entirely independent of this symbol or operation.

The first attempt to give a geometric meaning to the expression $a + bi$ appears to be due to Wallis in 1685, who proposed to construct the imaginary roots of a quadratic by going out of the line on which they would have been laid off if real. In 1804 the Abbé Buée devised the now accepted representation by laying off the terms containing i as a factor, at right angles to the others, and showed how to add and subtract such expressions as $a + bi$. At about the same time Argand published independently the same idea, and still further developed it. The concept of a directed quantity as represented by an algebraic symbol was thus necessarily arrived at. Gauss, Cauchy, and others have elaborated the complex unit more especially in the theory of numbers,

while Euler, Peacock, De Morgan, and others have developed it more as a double algebra.

Up to this point i had been regarded as a scalar operator merely, and the corresponding geometry only plane, though attempts had been made without much success to extend the treatment into three-dimensional space. It remained for Hamilton to accomplish this by the simple device of making i a directed operator, or handle, perpendicular to the plane of rotation, which opened the way for any number of similar operators differing in direction, but, as to their other properties, simply square roots of minus one. In order to produce a convenient algebra on this basis, Hamilton was obliged to take the further step of giving to all vectors the properties of $\sqrt{-1}$, and thus the calculus of quaternions was produced, a non-commutative quadruple algebra. These ideas have been generalized still farther by Unverzagt in his "Theorie der goniometrischen und der longimetrischen Quaternionen." In this book the author first develops a trigonometry based on a general instead of a right-angled triangle, and then shows that the

operator $j = (-1)^{\frac{\lambda}{2}}$ (in which λ is the fundamental angle, taking the place of $\frac{\pi}{2}$) takes in this trigonometry the place of i in De Moivre's theorem generalized. He then takes three units j_1, j_2, j_3 , corresponding to Hamilton's i, j, k , and forms a generalized quaternion, based on some angle λ , which reduces to the ordinary system when $\lambda = \frac{\pi}{2}$. The case particularly discussed is that in which $\lambda = 0$.

The theory and laws of linear, associative algebras, which includes quaternions as a particular case, have been thoroughly treated by Peirce in his work bearing that title.

We turn now to the other line along which multiple algebras have been developed. In 1827 Möbius published his "Barycentrische Calcul," in which points are the ultimate units, to which any desired weights may be assigned. He gave the laws of combination of these units so far as addition and subtraction are concerned, but did not proceed to multiplication: in fact, he distinctly states that they can be multiplied only by numbers. He then proceeds to treat analytical geometry on this basis. His treatment of points, so far as it goes, is on the same plan afterwards independently developed by Grassmann.

In 1844, one year after Hamilton's first announcement of his discovery, Grassmann published his "Ausdehnungslehre," which contains a complete and logical exposition of his new algebra for any number of independent units, and hence, geometrically interpreted, for space of any dimensions. This book was so abstract and general in form, and so unlike the ordinary language of mathematics, that it attracted hardly any notice, and the author was obliged to recast and republish it in 1862. Grassman's algebra is non-linear, and only partially associative, so that it differs fundamentally from all those discussed by Peirce. The $\sqrt{-1}$ plays no part whatever in the theory, and Grassman's vector is a vector pure and simple, i.e., a quantity having direction and magnitude, and not, as in quaternions, a *versor*-vector, combining the properties of a vector and of the $\sqrt{-1}$. The fundamental notion of Grassmann's multiplication is extension or generation; the product $p_1 p_2$ is the line generated by a point moving straight from p_1 to p_2 , etc.

In this great invention of Grassman we have a multiple algebra which is the natural language of geometry and mechanics, dealing in a manner astonishingly simple, concise, and expressive with these subjects, and certain, it appears to me, to gain constantly in the appreciation of math-

ematicians as it is more generally understood and used. The fact of its perfect adaptability to n -dimensional space is an additional argument in its favor for those who are interested in that line of investigation.

We have now traced the development of our subject from its elementary beginnings through a long period in which it was in the rhetorical stage, approaching at intervals here and there to the syncopated; then, on the revival of learning in Europe after the dark ages, we have seen its comparatively rapid progress through the syncopated stage to the purely symbolical, when it was at last in a shape suitable for the astonishing progress of the last two hundred years. Finally, in the present century, we have noted the appearance, as in the fulness of time, of multiple algebras from different and independent sources, whose realm is that of the future.

NOTES AND NEWS.

THE astronomers sent to the Sandwich Islands recently on the part of the International Geodetic Association of Europe and the United States Coast and Geodetic Survey, in order to make a more exhaustive study of the changes of latitude, have located their observatories at Waikiki, near Honolulu. It is proposed to observe during the year about sixty-five pairs of stars, chosen on account of their well-determined proper motions, and to make in all not far from twenty-five hundred observations of the latitude. The results, compared with those made simultaneously in Europe and America, will settle definitely the question whether there is a real motion of the pole. At the suggestion of the American representative, the force of gravity will be measured every night that latitude observations are made. This may throw light on one of the theories proposed to explain the changes of latitude, viz., that of large transfers of matter beneath the earth's surface. The new pendulums made at the Coast and Geodetic Survey Office in Washington, and which are similar to those taken to Alaska by Professor Men-denhall last spring, will be employed at Waikiki. They are of fine workmanship, and are capable of detecting changes that do not exceed one hundred-thousandth part of the quantity measured. Besides the observations at the regular station, a number of magnetic determinations will be made at other points in the Islands, — notably at Kealakeakua Bay, where Captain Cook observed the declination more than a hundred years ago, and at Lahaina, where De Freycinet had an observatory for pendulum and magnetic work in 1819. The re-occupation of these points will show the change of the needle during the past century, and will be of great value in determining the secular variation. It is intended also to seize the opportunity now presented to measure the force of gravity on the summit of Mauna Kea (14,000 feet elevation). Observations made at the top of Haleakala (10,000 feet) in 1887 showed conclusively that the mountain was solid. This fact received additional support from the zenith observations at the sea-level north and south of the mountain. The large deviation of the plumb line (29") brought to light in that work has now been exceeded on Hawaii, where 1' 26" has been discovered at the south point of the island (Ka Lae). This fact, recently communicated by Surveyor-General Alexander, makes the question of the force of gravity at the summit of Mauna Kea one of double interest, and it is desirable, both from a geological and geodetic standpoint, that pendulum observations be made on top of one of the mountains. Doctor Marcuse, who is from the Royal Observatory at Berlin, observes for latitude on the part of the European association, and Mr. Preston, who made the observations at the summit of Haleakala four years ago, is from the United States Coast and Geodetic Survey, and makes gravity and magnetic determinations. He also, as the representative of the United States, observes for latitude in connection with Dr. Marcuse, in the international geodetic work. The observers had the good fortune to arrive at Honolulu on the day preceding the transit of Mercury (9th of May), and made successful observations of the phenomenon. The second contact was also observed by Mr. Lyons of the government survey. The two interior contacts were noted by local mean time (Waikiki 8° east of Honolulu) as follows: —

	H.	M.	S.	H.	M.	S.
Mr. Lyons.....	1	26	32	—	—	—
Mr. Preston.....		26	53	6	10	50
Dr. Marcuse.....		27	3	11	22	

The station was in latitude 21° 16' 21" north, and in longitude 157° 49' 30" west. The mean observed times of contact are in both cases about a minute less than the computed ones.

SCIENCE:

A WEEKLY NEWSPAPER OF ALL THE ARTS AND SCIENCES

PUBLISHED BY

N. D. C. HODGES,

47 LAFAYETTE PLACE, NEW YORK.

SUBSCRIPTIONS.—United States and Canada.....\$3.50 a year.

Great Britain and Europe..... 4.50 a year.

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THE EVOLUTION OF THE CEREBELLUM.

THE cerebellum has unquestionably given more trouble to anatomists than almost any other organ, and our present knowledge of its structure seems disproportionate to the labor expended. It is no discredit to the monumental works of Stilling, Meynert, Purkinje, Gerlach, and Kölliker to admit that scarcely a single tract connecting the cerebellum with other portions of the brain is traced with sufficient detail. Even the external configuration of the cerebellum in lower animals has many lessons for us which may be useful in the interpretation of the human organ.

The cerebellum is subject to a greater range of variation than any other organ of the brain. From being practically absent, as in amphibia, to preponderating over all other segments of the brain in some fishes, there is every gradation in development. It becomes obvious from a brief study of the relative development of the regions of the encephalon that the cerebellum does not vary in proportion to the intelligence; that, in other words, it cannot be employed as a criterion of the position of the animal in the scheme of classification as can the cerebrum. Although not available for taxonomic purposes, these variations are none the less interesting from the clew which they may afford to the functions and laws of development of this and associated organs.

In the March number of the *Journal of Comparative Neurology* the writer called attention to the architectural modifications of the cerebellum in reptiles, and the progressive evolution of the organ in the several groups, as well as the resemblance of this course of evolution to a peculiar and apparently undescribed law of development of the cerebellum in mammals.

The cerebellum is peculiarly mobile, considered architecturally, by reason of its mode of attachment to the axis of the brain. It is morphologically the roof of the fourth ventricle. Both before and behind it is connected with the dorsal surface of the brain-tube by a velum, or thinned lamina,

devoid of nervous matter, and extensively folded and combined with vascular sinuses to form a nutritive organ, the plexus choroideus (metaplexus and mesaplexus). The velum posterior extends about the sides of the cerebellum also, so that rigidity is given to that organ only by the several fibre-bundles or peduncles of the cerebellum which connect it laterally with other regions. Thus, however large and heavy it may be, the cerebellum is supported solely by a lateral axis entering at the base. There is really nothing to prevent the most extensive rotations or foldings of the body in all directions except laterally.

The progressive development of this region is nowhere more conveniently illustrated than in the reptiles. Taking the transverse bar which constitutes the cerebellar rudiment in amphibians as a point of departure, we first encounter a leaf-like body with the ventricular half of the substance composed of granular material. The tracts are chiefly scattered in the dorsal white layer. In the serpents this flap is flexed so as to form a hood-like body. The flexure is due to the so-called pons-flexure of the whole medulla. The flexure is more pronounced in turtles, and results in a complete roof over the fourth ventricle, which may be considerably arched. It is obvious that there must be a limit to the development along this line. In higher reptiles, whose motions are more active and require more accurate co-ordination, the increase in size necessary to supply sufficient nervous matter renders necessary a complete eversion of the leaf-like organ. In the lizards the lamina is folded forward in such a way as to make a double roof over the ventricle, bringing the granular layer, with its neuro-epithelium dorsad, in the superior layer, while it faces ventrad (toward the ventricle) in the ventral lamina. In the alligators the development is more extensive, and results in a horizontally placed hollow cone, with the apex directed caudad, and attached by the ventral portion of its base to the brain base. The outside of the cone is clothed with epithelium, while the inside is the morphologically ectal surface. Of course, in this description the thin velum which originally connected the edges of the leaf has been disregarded. This eversion of the cerebellum is of the highest importance in preparing us to understand the origin of the cellular elements in the human cerebrum. Before alluding to this subject we may pass in review a few illustrations of cerebellar architectonic from other classes of vertebrates.

In fishes the range of variation is remarkable, in so far that it may render the brains of closely allied genera very dissimilar in appearance. The characteristics of the fish cerebellum, which serves to distinctly separate it from all other classes of mammals, is the development of a second portion of the organ in front of the valve of Vieussens, which is the morphological anterior (cephalad) margin of the cerebellum in other cases. The relatively large amount of cerebellar substance required by active fishes, and the lack of definite walls to the cephalic cavity, result in curious folds on a large scale and simple plan. The forward fold in front of the valve, which the writer has termed volvula, from its purse-like form, often completely fills the cavity of optic lobes, and in some cases (as the black-horse, *Cycleptus*), actually pries the two halves of the roof or tectum of that organ apart, and protrudes upon the dorsal surface with only the membranous velum tecti above it. The moon-eye, *Hyodon*, is the most reptilian of the osseous fishes so far examined, and in this case the cerebellum proper is a simple sac extending caudad: there is no external evidence of a volvula or of lateral lobes or "bursa." Sections show,

nevertheless, that there is a small volvula which lies, as in *Lepidosteus*, in the posterior part of the optic ventricle.

One curious result of the development of a volvula is the peculiar course by which the fourth cranial nerve reaches its centre. Entering at the usual place in the valve, it has to traverse a large part of the volvula before making its exit from the brain.

In the drum (*Haplodonotus*) the brain as a whole is exceedingly short. This shortening has the effect to tilt the optic lobes and cerebellum at a considerable angle with the axis of the brain, and to roll the volvula into a spherical mass with three folds, which are packed closely into the cavity of the ventricle. The main lobe of the cerebellum also has a short cephalad spur.

In the cat-fish family the cerebellum, instead of projecting backward, is thrust cephalad, affording a very good and constant differential character. The few illustrations here cited are derived from a memoir about to appear in the *Journal of Comparative Neurology*, where a wider range of comparison and full illustration may be possible.

It will be noticed that in the above cases the gray or granular material is ental. It has been shown by Professor His that the nervous elements in the spinal cord and medulla arise from the ventricular epithelium. This the writer has shown is also the case in the cerebellum, at least in reptiles and fishes. In the massive cerebellum of mammals we are struck by the difficulty which stands in the way of the carrying out of the same fundamental plan of structure. The active cells are separated from the epithelium by impervious masses of fibres. How, then, do these cells reach their destination? This important question we at first sought to solve by discovering in some embryonic stage an eversion similar to that described in reptiles. This proved to be a valuable clew, but not actually correct, though a tendency to revolve from behind forward is very pronounced in the cerebellum of birds, and is exhibited in the direction of the lobules of the cerebellum in marsupials. But, while there is not an actual eversion of the cerebellum in mammals, there is a time when a pouch from the lateral posterior walls of the fourth ventricle is formed. This diverticle envelops the cerebellum and meets its fellow of the opposite side. In a short time this sac flattens out, and both layers fuse with the ectal surface of the cerebellum, and constitute a temporary proliferating organ from which the cells are derived. These cells migrate to a point beneath the layer of Purkinje's cells, the origin of which seems to be also from the ventricular epithelium. Although this process has been observed only in rodentia there can be no doubt that it prevails in other groups of mammals. Although somewhat unexpected, this method is not unlike that which Professor His has described for the origin of the olives and related structures of the medulla. By this provision the increase of ectal surface through the convolutions of the cerebellum provides for the largest possible enlargement of the active centres with the most economical distribution of fibres.

This discovery may serve to enforce the value of a comparative method in approaching a complicated problem like the present one.

C. L. HERRICK.

JOHN GILMER SPEED follows up his article in the September *Lippincott's* with a paper entitled "The Common Roads of Europe." He shows how far ahead of us the great nations of Europe are in the matter of roads and their administration and maintenance. Among other articles in the October number may be mentioned a paper by William Agnew Paton upon "The Lost 'Land-fall' of Columbus."

ANTHROPOLOGY PAST AND PRESENT.¹

[Continued from p. 172.]

It has been the custom to speak of the early Aryan, Semitic, and Turanian races as large swarms — as millions pouring from one country into another. It has been calculated that these early nomads would have required immense tracts of meadow land to keep their flocks, and that it was the search for new pastures that drove them, by an irresistible force, over the whole inhabitable earth.

This may have been so, but it may also have not been so. Anyhow, we have a right to suppose that, before there were millions of human beings, there were at first a few only. We have been told of late that there never was a first man; but we may be allowed to suppose, at all events, that there were at one time a few first men and a few first women. If, then, the mixture of blood by marriage and the mixture of language in peace or war took place at an early time, when the world was peopled by some individuals, or by some hundreds, or by some thousands only, think what the necessary result would have been. It has been calculated that it would only require six hundred years to populate the whole earth with the descendants of one couple, the first father being dolichocephalic and the first mother brachycephalic. They might, after a time, all choose to speak the Aryan language, but they could not choose their skulls, but would have to accept them from nature, whether dolichocephalic or brachycephalic.

Who, then, would dare at present to lift up a skull and say this skull must have spoken an Aryan language, or lift up a language and say this language must have been spoken by a dolichocephalic skull? Yet, though no serious student would any longer listen to such arguments, it takes a long time before theories that were maintained for a time by serious students, and were then surrendered by them, can be completely eradicated. I shall not touch to-day on the hackneyed question of the "home of the Aryans" except as a warning. There are two quite distinct questions concerning the home of the Aryans.

When students of philology speak of Aryans, they mean by Aryas nothing but people speaking an Aryan language. They affirm nothing about skulls, skins, hair, and all the rest. Arya with them means speakers of an Aryan language. While, on the contrary, students of physiology speak of dolichocephalic, orthognathic, euthycomic people, they speak of their physiological characteristics only, and affirm nothing whatever about language.

It is clear, therefore, that the home of the Aryas, in the proper sense of that word, can be determined by linguistic evidence only, while the home of a blue-eyed, blond-haired, long-skulled, fair-skinned people can be determined by physiological evidence only. Any kind of concession or compromise on either side is simply fatal, and has led to nothing but a promiscuous slaughter of innocents. Separate the two armies, and the whole physiological evidence collected by D'Omalius, D'Halloy, Latham, and their followers will not fill more than an octavo page; while the linguistic evidence collected by Benfey and his followers will not amount to more than a few words. Everything else is mere rhetoric.

The physiologist is grateful, no doubt, for any additional skull whose historical antecedents can be firmly established; the philologist is grateful for any additional word that can help to indicate the historical or geographical whereabouts of the unknown speakers of Aryan speech. On these points it is possible to argue. They alone have a really scientific value in the eyes of a scholar, because, if there is any difference of opinion on them, it is possible to come to an agreement. As soon, however, as we go beyond these mere matters of fact, which have been collected by real students, everything becomes at once mere vanity and vexation of spirit. I know the appeals that have been made for concessions and some kind of compromise between physiology and philology; but honest students know that on scientific subjects no compromise is admissible. With regard to the home of the Aryas, no honest philologist will allow himself to be driven one step beyond the statement that the unknown people who spoke Aryan languages were, at one time,

¹ Address before the section of Anthropology of the British Association for the Advancement of Science, at Cardiff, August, 1891, by Professor F. Max Müller, president of the section (*Nature*, Sept. 3).

and before their final separation, settled somewhere in Asia. That may seem very small comfort, but for the present it is all that we have a right to say. Even this must be taken with the limitations which, as all true scholars know, apply to speculations concerning what may have happened, say, five thousand or ten thousand years ago. As to the color of the skin, the hair, the eyes, of those unknown speakers of Aryan speech, the scholar says nothing; and when he speaks of their blood he knows that such a word can be taken in a metaphorical sense only. If we once step from the narrow domain of science into the vast wilderness of mere assertion, then it does not matter what we say. We may say, with Penka, that all Aryas are dolichocephalic, blue-eyed, and blond, or we may say, with Piétrement, that all Aryas are brachycephalic, with brown eyes and black hair (V. d. Gheyn, 1889, p. 26). There is no difference between the two assertions. They are both perfectly unmeaning. They are *vox et præterea nihil*.

My experiences during the last forty years have only served to confirm the opinion which I expressed forty years ago, that there ought to be a complete separation between philology and physiology. And yet, if I were asked whether such a divorce should now be made absolute, I should say, No. There have been so many unexpected discoveries of new facts, and so many surprising combinations of old facts, that we must always be prepared to hear some new evidence, if only that evidence is brought forward according to the rules which govern the court of true science. It may be that in time the classification of skulls, hair, eyes, and skin may be brought into harmony with the classification of language. We may even go so far as to admit, as a postulate, that the two must have run parallel, at least in the beginning of all things. But with the evidence before us at present, mere wrangling, mere iteration of exploded assertions, mere contradictions, will produce no effect on the true jury, which hardly ever consists of more than twelve trusty men, but with whom the final verdict rests. The very things that most catch the popular ear will by them be ruled out of court. But every single new word, common to all the Aryan languages, and telling of some climatic, geographical, historical, or physiological circumstance in the earliest life of the speakers of Aryan speech, will be truly welcome to philologists quite as much as a skull from an early geological stratum is to the physiologist, and both to the anthropologist, in the widest sense of that name.

But, if all this is so, if the alliance between philology and physiology has hitherto done nothing but mischief, what right, it may be asked, had I to accept the honor of presiding over this section of anthropology? If you will allow me to occupy your valuable time a little longer, I shall explain, as shortly as possible, why I thought that I, as a philologist, might do some small amount of good as president of the Anthropological Section.

In spite of all that I have said against the unholy alliance between physiology and philology, I have felt for years—and I believe I am now supported in my opinion by all competent anthropologists—that a knowledge of languages must be considered in future as a *sine quâ non* for every anthropologist.

Anthropology, as you know, has increased so rapidly that it seems to say now, "*Nihil humani a me alienum puto*." So long as anthropology treated only of the anatomy of the human body, any surgeon might have become an excellent anthropologist. But now, when anthropology includes the study of the earliest thoughts of man, his customs, his laws, his traditions, his legends, his religions, ay, even his early philosophies, a student of anthropology without an accurate knowledge of languages, without the conscience of a scholar, is like a sailor without a compass.

No one disputes this with regard to nations who possess a literature. No one would listen to a man describing the peculiarities of the Greek, the Roman, the Jew, the Arab, the Chinese, without knowing their languages, and being capable of reading the master-works of their literature. We know how often men who have devoted the whole of their life to the study, for instance, of Hebrew, differ, not only as to the meaning of certain words and passages, but as to the very character of the Jews. One authority states that the Jews, and not only the Jews, but all Semitic nations, were possessed of a monotheistic instinct. Another authority shows that all Semitic nations, not excluding the Jews, were polytheistic

in their religion, and that the Jehovah of the Jews was not conceived at first as the Supreme Deity, but as a national god only, as the God of the Jews, who, according to the latest view, was originally a fetish or a totem, like all other Gods.

You know how widely classical scholars differ on the character of Greeks and Romans, on the meaning of their customs, the purpose of their religious ceremonies—nay, the very essence of their gods. And yet there was a time, not very long ago, when anthropologists would rely on the descriptions of casual travellers, who, after spending a few weeks, or even a few years, among tribes whose language was utterly unknown to them, gave the most marvellous accounts of their customs, their laws, and even their religion. It may be said that anybody can describe what he sees, even though unable to converse with the people. I say, decidedly no; and I am supported in this opinion by the most competent judges. Dr. Codrington, who has just published his excellent book on the "Melanesians: their Anthropology and Folk-Lore," spent twenty-four years among the Melanesians, learning their dialects, collecting their legends, and making a systematic study of their laws, customs, and superstitions. But what does he say in his preface? "I have felt the truth," he says, "of what Mr. Fison, late missionary in Fiji, has written: 'When a European has been living for two or three years among savages, he is sure to be fully convinced that he knows all about them; when he has been ten years or so amongst them, if he be an observant man, he knows that he knows very little about them, and so begins to learn.'"

How few of the books in which we trust with regard to the characteristic peculiarities of savage races have been written by men who have lived among them for ten or twenty years, and who have learnt their languages till they could speak them as well as the natives themselves.

It is no excuse to say that any traveller who has eyes to see and ears to hear can form a correct estimate of the doings and sayings of savage tribes. It is not so, and anthropologists know from sad experience that it is not so. Suppose a traveller came to a camp where he saw thousands of men and women dancing round the image of a young bull. Suppose that the dancers were all stark naked, that after a time they began to fight, and that at the end of their orgies there were three thousand corpses lying about weltering in their blood. Would not a casual traveller have described such savages as worse than the negroes of Dahomey? Yet these savages were really the Jews, the chosen people of God. The image was the golden calf, the priest was Aaron, and the chief who ordered the sacrifice was Moses. We may read the thirty-second chapter of Exodus in a very different sense. A traveller who could have conversed with Aaron and Moses might have understood the causes of the revolt and the necessity for the massacre. But without this power of interrogation and mutual explanation, no travellers, however graphic and amusing their stories may be, can be trusted; no statements of theirs can be used by the anthropologists for truly scientific purposes.

From the day when this fact was recognized by the highest authorities in anthropology, and was sanctioned by some at least of our anthropological, ethnological, and folk lore societies, a new epoch began, and philology received its right place as the hand-maid of anthropology. The most important paragraph in our new charter was this, that in future no one is to be quoted or relied on as authority on the customs, traditions, and more particularly on the religious ideas of uncivilized races who has not acquired an acquaintance with their language sufficient to enable him to converse with them freely on these difficult subjects.

No one would object to this rule when we have to deal with civilized and literary nations. But the languages of Africa, America, Polynesia, and even Australia, are now being studied as formerly Greek, Latin, Hebrew, and Sanscrit only were studied. You have only to compare the promiscuous descriptions of the Hottentots in the works of the best ethnologists with the researches of a real Hottentot scholar like Dr. Hahn to see the advance that has been made. When we read the books of Bishop Callaway on the Zulu, of William Gill and Edward Tregear on the Polynesians, of Horatio Hale on some of the North American races, we feel at once that we are in safe hands, in the hands of real scholars. Even then we must, of course, remember that their knowledge of

the languages cannot compare with that of Bentley, or Hermann, or Burnouf, or Ewald. Yet we feel that we cannot go altogether wrong in trusting to their guidance.

I venture to go even a step further, and I believe the time will come when no anthropologist will venture to write on anything concerning the inner life of man without having himself acquired a knowledge of the language in which that inner life finds its truest expression.

This may seem to be exacting too much, but you have only to look, for instance, at the descriptions given of the customs, the laws, the legends, and the religious convictions of the people of India about a hundred years ago, and before Sanscrit began to be studied, and you will be amazed at the utter caricature that is often given there of the intellectual state of the Brahmans compared with what we know of it now from their own literature.

And if that is the case with a people like the Indians, who are a civilized race, possessed of an ancient literature, and well within the focus of history for the last two thousand years, what can be expected in the case of really savage races? One can hardly trust one's eyes when one sees the evidence placed before us by men whose good faith can not be questioned, and who nevertheless contradict each other flatly on the most ordinary subjects. We owe to one of our secretaries, Mr. Roth, a most careful collection of all that has been said on the Tasmanians by eye-witnesses. Not the least valuable part of this collection is that it opens our eyes to the utter untrustworthiness of the evidence on which the anthropologist has so often had to rely. In an article on Mr. Roth's book in *Nature*, I tried to show that there is not one essential feature in the religion of the Tasmanians on which different authorities have not made assertions diametrically opposed to each other. Some say that the Tasmanians have no idea of a Supreme Being, no rites or ceremonies; others call their religion Dualism, a worship of good and evil spirits. Some maintain that they had deified the powers of nature, others that they were Devil-worshippers. Some declare their religion to be pure monotheism, combined with belief in the immortality of the soul, the efficacy of prayers and charms. Nay, even the most recent article of faith — the descent of man from some kind of animal — has received a religious sanction among the Tasmanians. For Mr. Horton, who is not given to joking, tells us that they believed "they were originally formed with tails, and without knee-joints, by a benevolent being, and that another descended from heaven, and, compassionating the sufferers, cut off their tails, and with grease softened their knees."

I would undertake to show that what applies to the descriptions given us of the now extinct race of the Tasmanians applies with equal force to the descriptions of almost all the savage races with whom anthropologists have to deal. In the case of large tribes, such as the inhabitants of Australia, the contradictory evidence may, no doubt, be accounted for by the fact that the observations were made in different localities. But the chief reason is always the same — ignorance of the language, and therefore want of sympathy and impossibility of mutual explanation and correction.

Let me, in conclusion, give you one of the most flagrant instances of how a whole race can be totally misrepresented by men ignorant of their language, and how these misrepresentations are at once removed if travellers acquire a knowledge of the language, and thus have not only eyes to see, but ears to hear, tongues to speak, and hearts to feel.

No race has been so cruelly maligned for centuries as the inhabitants of the Andaman Islands. An Arab writer of the ninth century states that their complexion was frightful, their hair frizzled, their countenance and eyes terrible, their feet very large, and almost a cubit in length, and that they go quite naked. Marco Polo (about 1285) declared that the inhabitants are no better than wild beasts, and he goes on to say: "I assure you all the men of this island of Angamanain have heads like dogs, and teeth and eyes likewise; in fact, in the face they are just like big mastiff dogs."

So long as no one could be found to study their language, there was no appeal from these libels. But when, after the Sepoy mutiny in 1856, it was necessary to find a habitation for a large number of convicts, the Andaman Islands, which had already

served as a penal settlement on a smaller scale, became a large penal colony under English officers. The havoc that was wrought by this sudden contact between the Andaman Islanders and these civilized Indian convicts was terrible, and the end will probably be the same as in Tasmania — the native population will die out. Fortunately one of the English officers (Mr. Edward Horace Man) did not shrink from the trouble of learning the language spoken by these islanders, and, being a careful observer and perfectly trustworthy, he has given us some accounts of the Andaman aborigines which are real masterpieces of anthropological research. If these islanders must be swept away from the face of the earth, they will now, at all events, leave a good name behind them. Even their outward appearance seems to become different in the eyes of a sympathizing observer from what it was to casual travellers. They are, no doubt, a very small race, their average height being 4 feet 10½ inches. But this is almost the only charge brought against them which Mr. Man has not been able to rebut. Their hair, he says, is fine, very closely curled, and frizzly. Their color is dark, but not absolutely black. Their features possess little of the most marked and coarser peculiarities of the negro type. The projecting jaws, the prominent thick lips, the broad and flattened nose of the genuine negro, are so softened down as scarcely to be recognized.

But let us now hear what Mr. Man has to tell us about the social, moral, and intellectual qualities of these so-called savages, who had been represented to us as cannibals; as ignorant of the existence of a deity; as knowing no marriage, except what by a bold euphemism has been called communal marriage; as unacquainted with fire; as no better than wild beasts, having heads, teeth, and eyes like dogs — being, in fact, like big mastiffs.

"Before the introduction into the islands of what is called European civilization, the inhabitants," Mr. Man writes, "lived in small villages, their dwellings built of branches and leaves of trees. They were ignorant of agriculture, and kept no poultry or domestic animals. Their pottery was hand-made, their clothing very scanty. They were expert swimmers and divers, and able to manufacture well-made dug-out canoes and outriggers. They were ignorant of metals, ignorant, we are told, of producing fire, though they kept a constant supply of burning and smouldering wood. They made use of shells for their tools, had stone hammers and anvils, bows and arrows, harpoons for killing turtle and fish. Such is the fertility of the island that they have abundance and variety of food all the year round. Their food was invariably cooked, they drank nothing but water, and they did not smoke. People may call this a savage life. I know many a starving laborer who would gladly exchange the benefits of European civilization for the blessings of such savagery."

These small islanders, who have always been represented by a certain class of anthropologists as the lowest stratum of humanity, need not fear comparison, so far as their social life is concerned, with races who are called civilized. So far from being addicted to what is called by the self-contradictory name of communal marriage, Mr. Man tells us that bigamy, polygamy, polyandry, and divorce are unknown to them, and that the marriage contract, so far from being regarded as a merely temporary contract, to be set aside on account of incompatibility of temper or other such causes, is never dissolved. Conjugal fidelity till death is not the exception but the rule, and matrimonial differences, which occur but rarely, are easily settled with or without the intervention of friends. One of the most striking features of their social relations is the marked equality and affection which exist between husband and wife; and the consideration and respect with which women are treated might, with advantage, be emulated by certain classes in our own land. As to cannibalism and infanticide, they are never practised by them.

It is easy to say that Mr. Man may be prejudiced in favor of these little savages, whose language he has been at so much pains to learn. Fortunately, however, all his statements have lately been confirmed by another authority, Colonel Cadell — the chief commissioner of these islands. He is a Victoria Cross man, and not likely to be given to over much sentimentality. Well, this is what he says of these fierce mastiffs, with feet a cubit in length:

"They are merry little people," he says. "One could not im-

agine how taking they were. Everyone who had to do with them fell in love with them [these fierce mastiffs]. Contact with civilization had not improved the morality of the natives, but in their natural state they were truthful and honest, generous and self-denying. He had watched them sitting over their fires cooking their evening meal, and it was quite pleasant to notice the absence of greed and the politeness with which they picked off the tit-bits and thrust them into each other's mouths. The forest and sea abundantly supplied their wants, and it was therefore not surprising that the attempts to induce them to take to cultivation had been quite unsuccessful, highly as they appreciated the rice and Indian corn which were occasionally supplied to them. All was grist that came to their mill in the shape of food. The forest supplied them with edible roots and fruits. Bats, rats, flying foxes, iguanas, sea-snakes, mollusks, wild pig, fish, turtle, and last, though not least, the larvæ of beetles, formed welcome additions to their larder. He remembered one morning landing by chance at an encampment of theirs, under the shade of a gigantic forest tree. On one fire was the shell of a turtle, acting as its own pot, in which was simmering the green fat delicious to more educated palates; on another its flesh was being broiled, together with some splendid fish; on a third a wild pig was being roasted, its drippings falling on wild yams, and a jar of honey stood close by, all delicacies fit for an alderman's table."

These are things which we might suppose anybody who has eyes to see, and who is not wilfully blind, might have observed. But when we come to traditions, laws, and particularly to religion, no one ought to be listened to as an authority who cannot converse with the natives. For a long time the Mincopies have been represented as without any religion, without even an idea of the Godhead. This opinion received the support of Sir John Lubbock, and has been often repeated without ever having been re-examined. As soon, however, as these Mincopies began to be studied more carefully, — more particularly as soon as some persons resident among them had acquired a knowledge of their language, and thereby a means of real communication, — their religion came out as clear as daylight. According to Mr. E. H. Man, they have a name for God — *Pûluga*. And how can a race be said to be without a knowledge of God if they have a name for God? *Pûluga* has a very mythological character. He has a stone house in the sky; he has a wife, whom he created himself, and from whom he has a large family, all except the eldest being girls. The mother is supposed to be green (the earth?), the daughters black; they are the spirits, called *Môrowin*; his son is called *Pijchor*. He alone is permitted to live with his father, and to convey his orders to the *Môrowin*. But *Pûluga* was a moral character also. His appearance is like fire, though nowadays he has become invisible. He was never born, and is immortal. The whole world was created by him, except only the powers of evil. He is omniscient, knowing even the thoughts of the heart. He is angered by the commission of certain sins, — some very trivial, at least to our mind, — but he is pitiful to all who are in distress. He is the judge from whom each soul receives its sentence after death.

According to other authorities, some Andamanese look on the sun as the fountain of all that is good, the moon as a minor power; and they believe in a number of inferior spirits, — the spirits of the forest, the water, and the mountain, — as agents of the two higher powers. They believe in an evil spirit also, who seems to have been originally the spirit of the storm. Him they try to pacify by songs, or to frighten away with their arrows.

I suppose I need say no more to show how indispensable a study of language is to every student of anthropology. If anthropology is to maintain its high position as a real science, its alliance with linguistic studies cannot be too close. Its weakest points have always been those where it trusted to the statements of authorities ignorant of language and of the science of language. Its greatest triumphs have been achieved by men such as Dr. Hahn, Bishops Callaway and Colenso, Dr. W. Gill, and last, not least, Mr. Man, who have combined the minute accuracy of the scholar with the comprehensive grasp of the anthropologist, and were thus enabled to use the key of language to unlock the perplexities of savage customs, savage laws and legends, and, particularly, of savage

religions and mythologies. If this alliance between anthropology and philology becomes real, then, and then only, may we hope to see Bunsen's prophecy fulfilled, that anthropology will become the highest branch of the science for which this British Association is instituted.

Allow me in conclusion once more to quote some prophetic words from the address which Bunsen delivered before our section in 1847:

"If man is the apex of the creation, it seems right, on the one side, that a historical inquiry into its origin and development should never be allowed to sever itself from the general body of natural science, and in particular from physiology. But, on the other side, if man is the apex of the creation, if he is the end to which all organic formations tend from the very beginning, if man is at once the mystery and the key of natural science, if that is the only view of natural science worthy of our age, then ethnological philology (I should prefer to say anthropology), once established on principles as clear as the physiological are, is the highest branch of that science for the advancement of which this association is instituted. It is not an appendix to physiology or to anything else; but its object is, on the contrary, capable of becoming the end and goal of the labors and transactions of a scientific association."

Much has been achieved by anthropology to justify these hopes and fulfil the prophecies of my old friend Bunsen. Few men live to see the fulfilment of their own prophecies, but they leave disciples whose duty it is to keep their memory alive, and thus to preserve that vital continuity of human knowledge which alone enables us to see in the advancement of all science the historical evolution of eternal truth.

LETTERS TO THE EDITOR.

*** Correspondents are requested to be as brief as possible. The writer's name is in all cases required as proof of good faith.*

On request in advance, one hundred copies of the number containing his communication will be furnished free to any correspondent.

Communication with Other Planets.

I NOTICE a letter from Sir Robert Ball with reference to the recent bequest of a French lady of 20,000 francs for a method of signalling to other planets. He enumerates different methods, but he does not speak of one method which I have never seen discussed but which seems to me worthy of mention. On a moonless clear night the electric and gas lights over such a large territory as New York and its suburbs must present the appearance to a spectator in Mars of a spot of light on the dark side of our globe. If, now, on such a night, from the middle of some large dark area, for example, the Atlantic Ocean, brilliant flashes of light be regularly sent forth in certain forms, there would be a chance of its being interpreted by the inhabitants of other spheres, and they might thereby be induced to signal us in return.

But there is a bare possibility of direct communication by taking advantage of the meteor currents in the great ocean of space in which we move. If on breaking open a meteorite we should find a chipped flint or other instrument, we should conclude that the portion of space from which it came had intelligent inhabitants. If, now, we can by the aid of modern explosives project into some meteor shoal a ball of iron containing at its center some object of human design, the ball might ultimately come to some other planet and be found by its inhabitants. Such a procedure would be analogous to casting a bottle inclosing a message upon the ocean to be wafted by the currents to some intelligent eye.

The wild schemes of one generation are often achieved in the next. Jules Verne's "Around the World in Eighty Days" and "Twenty Thousand Leagues under the Sea" have been in a measure realized, and possibly his "Voyage to the Moon" is the next romance to be realized in some form. An initial velocity of seven miles a second would be required to project a body beyond the earth's attraction, and it is not too much to hope that this will soon be attainable at the present rate of progress in the science of explosives. A projectile sent from the earth would have considerable value as a direct astronomical experiment on meteorites, even if it should fail in bringing tidings from another planet.

HIRAM M. STANLEY.

Lake Forest, Ill., Sept. 26.

The Man of the Future.

MAN being, zoologically, the highest organism known, the question of his further evolution and its probable direction is one which is naturally of interest to the student of anthropology. Is the development of a higher species possible or probable?

In order to even partially answer this question we must first consider what would constitute perfection or an approach to it. We are met at the very threshold of the problem by the fact, apparent on every hand, that the human product of our present civilization is, in the vast majority of instances, a very unequally developed organism, physically and mentally: an asymmetrical creature, the victim, so to speak, of specialism of one kind or another; it may be of environment or of occupation, usually of both. Hence the necessarily great diversity of view as to what would be improvement in the race.

It is a self-evident fact that improvement of the individual means, in the long run, advancement of the race; but when we ask what constitutes individual improvement, we receive one answer from the sociologist, another from the political economist, a third from the athlete or artist, and still a different reply from the philosopher.

That an easily-governed and equably-tempered people are not necessarily of a high type of humanity is evident from the experience of China, where a population of 350,000,000 is governed by a standing army of only 80,000 and the simplest police regulations, — a state of affairs unparalleled in the history of the so-called civilized nations. The perfection of their civil service and the insignificant expense of state officials are also said to be marvellous in the light of our experience. This state of affairs has been attributed to the perfection of their social training system, but it seems to the writer also allowable to suppose that many centuries of low diet, with overcrowding and mental subjection, have largely eliminated that spirit of restlessness and opposition to authority so characteristic of western peoples.

The ideas of the gymnast or exponent of physical culture, and of the purely intellectual sophist, are equally at fault in the solution of our question of improvement of the race; for it is evident to all medical men, at least, that powerful muscles may be a source of vital weakness to their possessor by inducing heart strain, or by overtaxing the eliminative capacity of the kidneys and other depurative organs, in the disposal of the results, the ashes so to speak, of muscular combustion. And, again, even a powerful brain may be menaced in its integrity by a weak or diseased blood-vessel incapable of withstanding the pressure caused by high functional activity.

The zoologist, therefore, sees the true strength of the individual, the race, and the species in (1) their plasticity, i.e., adaptability or capacity for modification in response to changes of environment, provided this plasticity be not the cause of localized weaknesses; and (2) equilibrium of development, organic balance, so to speak, between the component parts of the animal.

The question has been asked, Would not the development of wings or other additional organs be an improvement in the species? The reply to this, on zoological principles, would be No. The addition of wings, for instance, would be a retrogression to a lower type; and if the wings were feathered, a still further retrogression. We must conclude, then, that wings are incompatible with progress, zoologically considered, all visions of angels to the contrary notwithstanding.

In fact it is difficult to foresee any structural change of plan that would be an improvement under our present environment.

While, therefore, we may anticipate an increase in the average perfection of parts, and consequently a more harmonious development of man's present plan of structure, we cannot rationally look for any radical change in the plan itself. Hence we may conclude, upon purely theoretical grounds, as well as zoological experience, that under present conditions man is, in his plan of structure, the highest type of animal that can be produced; and while this may seem to coincide with the philosophy of the fatalist, "whatever is, is right," it is nevertheless true.

Improvement in organic balance or correlation of organs, therefore, is the chief direction of advance possible, and this improvement may be favored by the avoidance of excessive specialism in

environment and occupation, by which certain organs are overdeveloped, while others are dwarfed from disuse.

The ideal man is the perfectly balanced man, physically and mentally; neither an athlete, an intellectual giant, nor a zoological monstrosity, but a being harmoniously developed in all parts.

It may be objected that such an organism would, like the "one-hoss shay" of Dr. Holmes, go to pieces all at once. But would not this imply a useful capacity and activity up to a good old age, of which we have striking examples in the lives of Audubon, Jefferson, and Darwin, and be vastly preferable to a break at forty from some one weak organ?

The maxim, "A chain is only as strong as its weakest link," applies here as forcibly as it does in mechanics.

DR. F. W. LANGDON.

Cincinnati, O., Sept. 15.

BOOK-REVIEWS.

A History of Chemistry. By ERNST VON MEYER. Translated by George M'Gowan. New York and London, Macmillan. 8°. \$4.50.

THERE have not been many histories of chemistry written, the best known being probably that of Kopp, which began to appear nearly fifty years ago. The same is true of physics, the only one that now occurs to us being that of Poggendorf, yet it has often seemed that it would be well for the student to be informed of the way by which the present state of science has been reached, yet it is but rarely that such information is imparted in lectures or in the class-room.

One reason for this may be in the worthlessness of what has gone before, which is strongly brought out in our author's treatment of alchemy. The benefits which accrued to chemistry during the centuries which were occupied by alchemists in the attempt to turn the baser metals into gold he estimates as very slight. It was but seldom that a discovery of technical value, like that of the making of porcelain, sprang from alchemistic work. On the other hand, positive harm was done by the tendency to mysticism among many of the workers, which resulted frequently in deliberate fraud.

As man seems really to care first for wealth and then for health, — at least he will give up the latter in pursuit of the former, — so after the attempt to gain riches through chemical knowledge had proved futile, attention was turned by investigators — and these of a higher class of intelligence than the earlier alchemists — to a development of the knowledge of drugs and their action on the human system, and to this period a chapter is devoted.

One phenomenon which was evidently chemical in its nature — combustion — had always interested chemists, and gave rise to the phlogiston theory, which held sway from soon after the death of Boyle till the discovery of oxygen in 1774. This theoretical view of combustion had a great influence on the progress in the accumulation of chemical truths, and it may be that here again some would urge that little attention need now be paid to these antiquated doctrines. But it may be well for the wisest of the physicists and chemists of the present day to occasionally have it pointed out to them how essentially false doctrines can maintain themselves for considerable periods when once they have received the endorsement of men wise in their day and generation.

Chemistry, as we know it, came into full possession of its faculties with the beginning of the use of the balance and the discovery of oxygen. The systematization of the facts as they were discovered was based, first, on the atomic theory of Dalton, and later on the doctrine of valency as developed during the past thirty years. All this development of the science is described historically in the early chapters of the book before us.

The closing chapter is devoted to a special history of the various branches of chemistry, from Lavoisier to the present day. In this chapter are treated the history of analytical chemistry, and of pure chemistry, inorganic and organic, which naturally leads to a consideration of thermo-chemistry or of physical chemistry in general. Technical chemistry is not neglected, nor are physiological and agricultural, which offer as difficult problems as any in the whole science. A section of interest is devoted to the methods of chemi-

cal instruction, which are attracting much attention in Great Britain and Germany.

We believe Meyer's "History of Chemistry," standing as it does alone, should be much used by teachers and students.

A Manual of the Steam Engine. Part I.: Structure and Theory.

By ROBERT H. THURSTON. New York, Wiley. 8°. \$7.50.

It has been a common slur at the thermo-organic theory of heat-engines that it has led to little of the improvement in their construction. It is well known that the thermo-dynamics of the steam-engine was not understood, or at least generally recognized, till the best part of a century had passed after the first introduction of the engine as a practical motive power. But even then the theory applied only to an ideal engine—an engine consisting of a few diagrammatic lines called a "hot body," a "cold body," etc., and known as the Carnot engine. That this theoretical explanation of the working of the heat engine is of the greatest interest, and has certainly aided materially in clearing engineers' ideas as to the possibilities of the heat-engine, cannot be denied; but there are so many differences between the Carnot engine and the steam-engine of practice that the improvement of the latter has been forced to depend on the "rule of thumb" for guidance.

Professor Thurston, in this book, makes an attempt to carry the theory forward a step and put it in such shape as to be applicable to the real engine. He does not maintain that this can as yet be done with perfect satisfaction, but only that sufficient knowledge

of the various wastes of heat has been obtained to justify this treatise. The subject is still obscure, but it is believed that the provisional theory and purposed processes of computation will aid the engineer materially in his endeavor to anticipate the performance of any new engine, the design of which may be hand.

Cotterill's "The Steam-Engine considered as a Heat-Engine" was perhaps an earlier attempt in the same direction, and with the progress of experimental work at the numerous technical institutions the world over, modifications in Thurston's treatment must be called for.

This first part covers the structure and theory of the steam-engine; the second part will be devoted to the design, construction, and operation.

AMONG THE PUBLISHERS.

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- BROOKLYN Institute, Third Year-Book of the (1890-91). Brooklyn, Eagle Pr. 232 p. 12°.
MERRILL, G. P. Stones for Building and Decoration. New York, Wiley. 453 p. 8°. \$5.
SCHUCHHARDT, C. Schliemann's Excavations: an Archaeological and Historical Study. New York, Macmillan. 363 p. 8°. \$4.
SMITH, E. F., and KELLER, H. F. Experiments Arranged for Students in General Chemistry. 2d ed. Philadelphia, Blakiston. 56 p. 8°.
SOLMS-LAUBACH, H. Graf zu, Fossil Botany. New York, Macmillan. 401 p. 8°. \$4.50.
THORNE R. T. Diphtheria: Its Natural History and Prevention. London and New York, Macmillan. 266 p. 12°. \$2.

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